From the Teacher: K. Evans

Class: AP Statistics

Period: 6

Assignment: Week 1

If turning in paper packet and work, make sure to include this header information on all pages!

From the Student:
Student Name
Teacher Name
Name of class
Period #
Assignment #

Distance Learning 2020 Week 1

AP Exam Update, Inference about comparing two means

Assignments are accessible in Microsoft Teams on Office 365. Work can also be submitted in Teams, which I highly encourage you to do if you are able to. You can contact Ms. Evans if you need help with Teams. You must write your name in pen on each page of your assignment.

The work in this packet is not officially due until 5/8/2020. However, I have broken down the work into daily chunks to help you manage your time. I encourage you to have the work from week 1 complete by 4/24/2020. New assignments for weeks 2 and 3 will be given that date.

My office hours are 1 pm – 3 pm, M–F. You can reach me through Remind (class code: @evans-stat), email (kevans@tusd.net) or chat on Teams. Please continue to check your email regularly.

Ms. Evans will be holding a half hour meeting on Microsoft Teams to talk about 10.2 and answer questions on **Tuesday 4/21**. Check in Teams in the posts or the calendar to find the exact time.

Week 1: Day 1 (turn in by 5/8/2020): Review information about the updates to the AP Statistics exam.

1. Read over the AP Statistics Exam Updates PowerPoint slides.
2. Write 1-2 thoughtful paragraphs about what this means for you, the rest of the year, and what questions you have. (**Assignment #1**)

*If turning in work on Teams, you can type up your answer and upload the file. Or, you can write your answer on binder paper and then upload a picture of it. Please write your name in pen on each page before you take a picture. Make sure your picture is clear and readable.

Week 1: Day 2-3 (turn in by 5/8/2020): Inference procedures involving comparing two means

Read over PowerPoint notes on 10.2 Comparing Means.
Also read the section in the book p.634 – 653.

Assignment #2 is p.654 #31, 37, 40, 43, 46, 47, 51, 54

Week 1: Day 4-5 (turn in by 5/8/2020): More Practice

Complete Unit 7 Progress Check (all parts) on college board AP Classroom.

Offline: Write up a summary of all the inference procedures involving means (8.3, 9.3, 10.2)

AP Statistics Exam Updates PowerPoint

AP STATISTICS

2020 AP Exam
Friday, May 22 @ 11:00AM PST

New Testing Schedule

Exam Start Times: Local times may vary depending on a student's geographic location.	Hawaii Time: 6:00 a.m. Alaska Time: 8:00 a.m. Pacific Time: 9:00 a.m. Mountain Time: 10:00 a.m. Central Time: 11:00 a.m. Eastern Time: 12:00 p.m. Greenwich Mean Time: 4:00 p.m.	Hawaii Time: 8:00 a.m. Alaska Time: 10:00 a.m. Pacific Time: 11:00 a.m. Mountain Time: 12:00 p.m. Central Time: 1:00 p.m. Eastern Time: 2:00 p.m. Greenwich Mean Time: 6:00 p.m.	Hawaii Time: 10:00 a.m. Alaska Time: 12:00 p.m. Pacific Time: 1:00 p.m. Mountain Time: 2:00 p.m. Central Time: 3:00 p.m. Eastern Time: 4:00 p.m. Greenwich Mean Time: 8:00 p.m.
Mon, May 11	Physics C: Mechanics	Physics C: Electricity and Magnetism	United States Government and Politics
Tues, May 12	Latin	Calculus AB Calculus BC	Human Geography
Wed, May 13	Physics 2: Algebra-Based	English Literature and Composition	European History
Thurs, May 14	Spanish Literature and Culture	Chemistry	Physics 1: Algebra-Based
Fri, May 15	Art History	United States History	Computer Science A
Mon, May 18	Chinese Language and Culture	Biology	Environmental Science
Tues, May 19	Music Theory	Psychology	Japanese Language and Culture Italian Language and Culture
Wed, May 20	German Language and Culture	English Language and Composition	Microeconomics
Thurs, May 21	French Language and Culture	World History: Modern	Macroeconomics
Fri, May 22	Comparative Government and Politics	Statistics	Spanish Language and Culture

New Test Format will be …

- 25 min FRQ
- 5 min submit Answer
- 15 min FRQ
- 5 min sumit answer

45 Minutes Total

What Technology may I use?



Phone
iPad
Tablet
Laptop
Desktop

If you know of students who have a need for technology or connectivity in order to take their AP Exam, please contact Mrs. Rotondi ASAP

College Board has to know before April 24.



TWO OPTIONS

1. Type your answers on your electronic device
2. Handwrite your answers, take a photo with your phone, and submit using your electronic device



25 Minute FRQ

Worth 55% of score

Free-response question 1: Multi-focus free-response question that will consist of similar components to traditional Statistics exam questions, assessing 2 or 3 of the course skill categories (including the inference skills within each category).

15 MINUTE FRQ

Worth 45% of score

Free-response question 2: Multi-focus free-response question that will consist of similar components to traditional Statistics exam questions, assessing 2 or 3 of the course skill categories (including the inference skills within each category).

Both questions may cover 2 or more of the following

- **Exploring Data** (Chapters 1-3, 12.2)
- **Sampling and Experimentation** (Chapter 4)
- **Probability and Simulations** (Chapters 5-7)
- **Inference** (Chapters 8-10)

Other Information

- There will not be an Investigative Task (a #6 question) on the 2020 AP Statistics Exam.
- As on a traditional AP Exam, students may require access to the AP Statistics Formula Sheet and should access and/or print it before the exam.
  - *Formatted different from our green sheet but the same, so can use your green sheets*
- Questions on the 2020 AP Statistics Exam are designed such that required calculations can be done with a pencil and paper, with no calculator (including one with graphical or statistical capabilities) required. However, use of a calculator is allowed and may be helpful. Simple ("four-function") calculators are freely available as apps for computers and phones (i.e. most or all internet-connected devices) and can be installed beforehand for use on the exam.

General Exam Features

- Like many college-level exams, this year's AP Exams will be open book/open note. Get tips for taking open book/open note exams.
- Students will be able to take exams on any device they have access to—computer, tablet, or smartphone. They will be able to either type and upload their responses or write responses by hand and submit a photo via their cell phone.
- In late April, information on how to access the testing system on test day, and video demonstrations so that students can familiarize themselves with the system will be released.
- College Board has free online classes/review videos on YouTube, organized by subject and Topic: https://www.youtube.com/user/advancedplacement

AP statistics Testing Dates

Friday, May 22, 2020 @ 11:00 AM
From your home on your device

Makeup Date
Friday, June 5, 2020 @ 1:00 PM
ONLY if you have a conflict with May 18
Must be validated by your school

We Have Five WEEKS (From Mon 4/20)



- Learn 10.2 (inference with difference of 2 means)
- Review the key concepts we have studied
- Take advantage of the Personal Progress Checks in AP Classroom
- Utilize multiple practice exams that mimic the current exam structure over the next six weeks

10.2 Comparing Two Means PowerPoint

10.2 Comparing Two Mean

What if we want to compare the mean of some quantitative variable for the individuals in Population 1 and Population 2?

Our parameters of interest are the population means μ_1 and μ_2. The best approach is to take separate random samples from each population and to compare the sample means.

Suppose we want to compare the average effectiveness of two treatments in a completely randomized experiment. We use the mean response in the two groups to make the comparison.

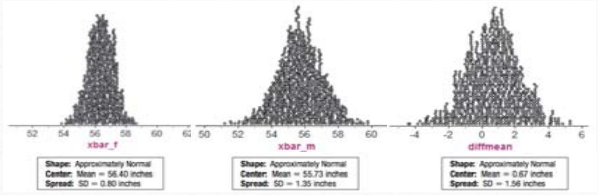
Population or Treatment	Parameter	Statistic	Sample Size
1	μ_1	$\bar{x}_1$	n_1
2	μ_2	$\bar{x}_2$	n_2

To explore the sampling distribution of the difference between two means, let's start with two Normally distributed populations having known means and standard deviations.

Based on information from the U.S. National Health and Nutrition Examination Survey (NHANES), the heights (in inches) of ten-year-old girls follow a Normal distribution $N(56.4, 2.7)$. The heights (in inches) of ten-year-old boys follow a Normal distribution $N(55.7, 3.8)$.

Suppose we take independent SRSs of 12 girls and 8 boys of this age and measure their heights.

Using Fathom software, we generated an SRS of 12 girls and a separate SRS of 8 boys and calculated the sample mean heights. The difference in sample means was then be calculated and plotted. We repeated this process 1000 times. The results are below:



Using what we know, lets describe the sampling distributions of $\bar{x}_1$, $\bar{x}_2$, and $\bar{x}_1 - \bar{x}_2$.

Answers for question on previous slide

Sampling Distribution of $\bar{x}_1$:

Shape: Normal; because the population distribution is normal

Center: $\mu = 56.4$ inches

Spread: $\sigma = \frac{\sigma}{\sqrt{n}} = \frac{2.7}{\sqrt{12}} = 0.78$ inches

Sampling Distribution of $\bar{x}_2$:

Shape: Normal; because the population distribution is normal

Center: $\mu = 55.7$ inches

Spread: $\sigma = \frac{\sigma}{\sqrt{n}} = \frac{3.8}{\sqrt{8}} = 1.34$ inches

Sampling Distribution of $\bar{x}_1 - \bar{x}_2$:

Shape: Normal; combining two independent Normal distributions

Center: $\mu = 0.7$ inches

Spread: $\sigma = 1.55$ inches,

When combining, always add variances to get new variance and square root to get new std. dev.

$\sqrt{0.78^2 + 1.34^2} = 1.55$

The Sampling Distribution of $\bar{x}_1 - \bar{x}_2$

Choose an SRS of size n_1 from Population 1 with mean μ_1 & standard deviation σ_1 and an independent SRS of size n_2 from Population 2 with mean μ_2 & standard deviation σ_2.

- **Shape:** When the population distributions are Normal, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is Normal. In other cases, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ will be approximately Normal if the sample sizes are large enough for CLT to apply (≥30)
- **Center:** The mean of the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is $\mu_1 - \mu_2$
- **Spread:** The standard deviation of the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is $\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}$ as long as each sample is no more than 10% of its population

Ex. A potato chip manufacturer buys potatoes from two different suppliers, Riderwood Farms and Camberley, Inc. The weights of potatoes from Riderwood Farms are approximately Normally distributed with a mean of 175 grams and a standard deviation of 25 grams. The weights of potatoes from Camberley are approximately Normally distributed with a mean of 180 grams and a standard deviation of 30 grams. When shipments arrive at the factory, inspectors randomly select a sample of 20 potatoes from each shipment and weigh them. Let $\bar{x}_C - \bar{x}_R$ be the difference in the sample mean weight of potatoes from the two suppliers.

a) What is the shape of the sampling distribution of $\bar{x}_C - \bar{x}_R$? Why?

b) Find the mean of the sampling distribution. Show your work.

c) Find the standard deviation of the sampling distribution. Show your work.

STOP

Answers for example on previous slide

a) *The shape of the sampling distribution of $\bar{x}_C - \bar{x}_R$ is approximately Normal because both population distributions are approximately Normal.*

b) *The mean is $\mu_{\bar{x}_C - \bar{x}_R} = 180 - 175 = 5$ grams.*

c) *Because 20 potatoes is less than 10% of each shipment, the standard deviation is $\sigma_{\bar{x}_C - \bar{x}_R} = \sqrt{\frac{30^2}{20} + \frac{25^2}{20}} = 8.73$ grams.*

The Two-Sample *t* Statistic

When data come from two independent random samples or two groups in a randomized experiment, the statistic $\bar{x}_1 - \bar{x}_2$ is our best guess for the value of $\mu_1 - \mu_2$.

When the 10% condition is met, the standard deviation of the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

If the Normal condition is met, we standardize the observed difference to obtain a *t* statistic that tells us how far the observed difference is from its mean in standard deviation units.

Since we don't know the values of the parameters σ_1 and σ_2, we replace them in the standard deviation formula with the sample standard deviations. The result if the standard error of the statistic $\bar{x}_1 - \bar{x}_2$:

$$SE_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

When we standardize the point estimate $\bar{x}_1 - \bar{x}_2$, the result is the two-sample t statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

The t statistic says how far $\bar{x}_1 - \bar{x}_2$ is from its mean in standard deviation units.

Conditions for Performing Inference about a Difference in Means

- **Random:** The data come from two independent random samples or from two groups in a randomized experiment.
- **10%:** When sampling without replacement, check that the 10% conditions is met for both populations.
- **Normal/Large Sample:** Both population distributions (or the true distributions of responses to the two treatments) are Normal **or** both sample sizes are large ($n_1 \geq 30$ and $n_2 \geq 30$) so CLT says they are approximately normal. **If** either population (treatment) distribution has unknown shape & the corresponding sample size is less than 30, use a graph of the sample data (both samples!) to assess the Normality of the population (treatment) distribution. Do not use two-sample t procedures if the graph shows strong skewness or outliers.

Degrees of Freedom with 2-sample procedures

There are two options for degrees of freedom with using the two-sample t procedures when conditions are met.

- **Technology:** Use the t-distribution with degrees of freedom calculated from the data by the formula below. The df from this formula is usually not a whole number.

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

- **Conservative:** Use the t distribution with degrees of freedom equal to the *smaller* of $n_1 - 1$ and $n_2 - 1$

Two-Sample t interval for the Difference between two Means

When the conditions are met, an appropriate C% confidence interval for $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where t^* is the critical value with C% of its area between $-t^*$ and t^* for t distribution with degrees of freedom from either technology or the smaller of $n_1 - 1$ and $n_2 - 1$.

Ex. Ashtyn and Olivia wanted to know if generic chocolate chip cookies have as many chocolate chips as name-brand chocolate chip cookies, on average. To investigate, they randomly selected 10 bags of Chips Ahoy cookies and 10 bags of Great Value cookies and randomly selected 1 cookie from each bag. Then, they carefully broke apart each cookie and counted the number of chocolate chips in each. Here are their results:

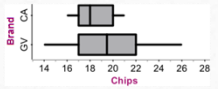
Chips Ahoy	17	19	21	16	17	18	20	21	17	18
Great Value	22	20	14	17	21	22	15	19	26	18

a) Construct and interpret a 99% confidence interval for the difference in the mean number of chocolate chips in Chips Ahoy and Great Value cookies.

b) Does your interval provide convincing evidence that there is a difference in the mean number of chocolate chips?

Answers for example on previous slide

a) *Populations: All Chips Ahoy cookies and Great Value cookies*
μ_{CA} = mean number of chocolate chips in Chips Ahoy cookies
μ_{GV} = mean number of chocolate chips in Great Value cookies
2-sample t interval for $\mu_1 - \mu_2$
Conditions: Random? Data came from two independent random samples
10%? There were more than 10(10)=100 Chips Ahoy cookies and more than 10(10)=100 Great Value cookies
Normal? Sample sizes are small, so must check graphs of data. The graphs show no obvious skewness or outliers, so it is safe to use t procedures.



$\bar{x}_{CA} = 18.4$, $s_{CA} = 1.78$, $\bar{x}_{GV} = 19.4$, $s_{GV} = 3.60$, *using df = 9, t* = 3.250*

$$(18.4 - 19.4) \pm 3.250\sqrt{\frac{1.78^2}{10} + \frac{3.60^2}{10}}$$

$$(18.4 - 19.4) \pm 3.250(1.2699)$$

CI: $(-5.13, 3.13)$ *Using df = 13.145, CI:* $(-4.81, 2.81)$

We are 99% confident that the interval from −4.81 to 2.81 captures the true difference in the mean number of chocolate chips in Chips Ahoy and Great Value cookies.

b) *Because the interval includes 0, there is not convincing evidence that there is a difference in the mean number of chocolate chips in Chips Ahoy and Great Value chocolate chip cookies.*

An observed difference between two sample means can reflect an actual difference in the parameters, or it may just be due to chance variation in random sampling or random assignment. Significance tests help us decide which explanation makes more sense.

The null hypothesis has the general form

$$H_0: \mu_1 - \mu_2 = \text{hypothesized value}$$

We're often interested in situations in which the hypothesized difference is 0. Then the null hypothesis says that there is no difference between the two parameters:

$$H_0: \mu_1 - \mu_2 = 0 \text{ or, alternatively, } H_0: \mu_1 = \mu_2$$

The alternative hypothesis says what kind of difference we expect.

$$H_a: \mu_1 - \mu_2 > 0, \; H_a: \mu_1 - \mu_2 < 0, \text{ or } H_a: \mu_1 - \mu_2 \neq 0$$

To do a test, standardize $\bar{x}_1 - \bar{x}_2$ to get a two-sample t statistic:

$$\text{test statistic} = \frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

To find the P-value, use the t distribution with degrees of freedom given by technology (more accurate df) or by df = smaller of $n_1 - 1$ *and* $n_2 - 1$ (conservative df).

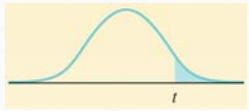
Two-Sample *t* test for the Difference between two Means

Suppose the conditions are met. To test the hypothesis $H_0: \mu_1 - \mu_2 =$ hypothesized value, compute the two-sample *t* statistic

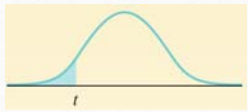
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Find the *p*-value by calculating the probability of getting a *t* statistic this large or larger in the direction specified by the alternative hypothesis H_a. Use the *t* distribution with degrees of freedom approximated by technology or the smaller of $n_1 - 1$ *and* $n_2 - 1$.

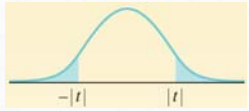
$H_a: \mu_1 - \mu_2 >$ hypothesized value



$H_a: \mu_1 - \mu_2 <$ hypothesized value



$H_a: \mu_1 - \mu_2 \neq$ hypothesized value



Ex. After buying many helium balloons only to see them deflate within a couple of days, Erin and Jenna decided to test if helium-filled balloons deflate faster than air-filled balloons. To find out, they bought 60 balloons and randomly divided them into two piles of 30, filling the balloons in the first pile with helium and the balloons in the second pile with air. Then, they measured the circumference of each balloon immediately after being filled and again three days later. The average decrease in circumference of the helium-filled balloons was 26.5 cm with a standard deviation of 1.92 cm. The average decrease of the air-filled balloons was 2.1 cm with a standard deviation of 2.79 cm.

a) Do these data provide convincing evidence that helium-filled balloons deflate faster than air-filled balloons?

b) Interpret the P-value you got in part (a) in the context of this study.

Answers for example on previous slide

a) *Populations: Balloons filled with helium and air*
μ_H = the mean decrease in circumference of helium-filled balloon after 3 days
μ_A = the mean decrease in circumference of air-filled balloon after 3 days
$H_0: \mu_H - \mu_A = 0$ and $H_a: \mu_H - \mu_A > 0$
2-sample t test for $\mu_H - \mu_A$
Conditions: Random? Data comes from two groups in a randomized experiment
10%? Not necessary to check since no sampling was done
Normal? Both samples (30 for both) are at least 30, so CLT says sampling distribution of $\bar{x}_H - \bar{x}_A$ is approximately normal.

$$t = \frac{26.5 - 2.1}{\sqrt{\frac{1.92^2}{30} + \frac{2.79^2}{30}}} = 39.46$$ *(include picture with shading)*

With df = 29 or even df = 51.4, p-value = P(t > 39.46) ≈ 0
Because the P-value of approximately 0 is less than α = 0.05, we reject H_0. There is convincing evidence that helium-filled balloons deflate faster than air-filled balloons.

b) *Assuming that the mean decrease in circumference is the same for helium-filled and air-filled balloons, there is an approximately 0 probability of getting a difference of 24.4 cm or more by chance alone.*

Using Two-Sample *t* Procedures Wisely

- ✓ In planning a two-sample study, choose equal sample sizes if you can.
- ✓ Do not use "pooled" two-sample t procedures!
- ✓ We are safe using two-sample t procedures for comparing two means in a randomized experiment.
- ✓ Do not use two-sample t procedures on paired data!
- ✓ Beware of making inferences in the absence of randomization. The results may not be generalized to the larger population of interest.

