

From the Teacher: J. Backster

Class: Algebra 1

Periods: 3,4,5

Assignment: Weeks 2 and 3

If turning in paper packet and work, make sure to include this header information on all pages!

From the Student:

Student Name

Teacher Name

Name of class

Period #

Assignment #

Distance Learning 2020 Weeks 2 and 3

This PDF has the assignments for Weeks 2 and 3. Do not go to the school to pick up this packet. You do not have to print this packet. Just write the problems and solutions on binder paper and text me pictures of your work through the Remind App. You must write your name in pen on each page of your assignment. Follow the labeling shown above.

ONLY If you can't take pictures of your work and can't send them to me digitally will you need to go to West High on April 24 between 9:30 am and 4 p.m to get this packet.

The work in this packet is not officially due until 5/15/2020.

My office hours are 10 am – 12 pm, M–F. You can reach me through texts on the Remind App.

Week 2

Graphing quadratic functions in Standard Form & Zeros of a function

Week 2: Day 1 (turn in by 5/8/2020): Graphing Quadratic Functions in Standard Form

Read over notes on Graphing quadratic functions in standard form.

Assignment #1 is Standard Form worksheet (see page 4 of this document)

Other resources that can help are

Finding the axis of symmetry and the vertex

<https://youtu.be/iKt6vjAygLc>

<https://youtu.be/5hQqj8EHNqo>

Week 2: Day 2–3 (turn in by 5/8/2020): Solve Quadratic equations graphically

Read over notes on Solving quadratic equations graphically. Can also read the book, Explain 1 in 20.1 on p.938-939.

Assignment #2 is p.945 #3-10 (page 5 and 6 of this document)

Other resources that can help are

<https://youtu.be/reRSfNfmcsk> (Sound isn't very loud, but good content)

Week 2: Day 4 (turn in by 5/8/2020): Factored form of a quadratic equation

Read over notes on Factored form of a quadratic equation. Can also read the book, Explain 1 & 2 in 20.2 on p.952-953

Assignment #3 is p.958 #5-14 (page 7 of this document)

Other resources that can help are

Rewriting in standard form

https://youtu.be/uFBbdMh2k_E

<https://youtu.be/gVracHjxQyM>

Week 2: Day 5 (turn in by 5/8/2020): Zero Product property

Read over notes on Zero Product Property. Can also read the book, Explore and Explain 1 in 20.3 on p.961-962.

Assignment #4 is p.966 #1-8 (page 8 of this document)

Other resources that can help are

On [Khan Academy](#)

<https://youtu.be/yCcMCPHFrVc>

Week 3

Factoring Trinomials and solving by factoring

Week 3: Day 1-2 (turn in by 5/15/2020): Factoring Trinomials

Read over notes on Factoring Trinomials

Assignment #1 is p.992 #3-8 and p.1004 #1-6 (pages 9 and 10 of this document)

Other resources that can help are

Factoring Common Factor out:

On [Khan Academy](#) (Three videos in a row that might help)

<https://youtu.be/EDebmfT5Nsk>

Factoring Trinomials using box method:

On [Algeomulus Prep Academy](#) (West High student made!)

Another text description: <https://www.basic-mathematics.com/factoring-using-the-box-method.html>

<https://youtu.be/d8MjmwHV-84>

<https://youtu.be/SWtQGRNKHOU>

Week 3: Day 3-4 (turn in by 5/15/2020): Solve Quadratic equations by factoring

Read over notes on Solving quadratic equations by factoring.

Assignment #2 is p.1005 #9-14 (page 11 of this document)

Other resources that can help are

On [Algeomulus Prep Academy](#) (West High student made!)

Week 3: Day 5 (turn in by 5/15/2020): Using Special Factors to solve equations

Read over notes on Using special factors to solve equations.

Assignment #3 is Special Cases worksheet (see page 4 of this document)

Other resources that can help are

<https://youtu.be/WPDaiXYmRoQ>

On [Algeomulus Prep Academy](#) (West High student made!)

<https://youtu.be/Zz1UjoentGk>

<https://youtu.be/EGzt8twijXc>

<https://youtu.be/HLNSouzygw0>

Standard Form (Week 2 Assignment #1)**Algebra 1**

Give the axis of symmetry and the coordinates of the vertex of the quadratic function.

1. $y = 2x^2 + 4x + 6$

2. $y = -3x^2 + 6x - 2$

3. $y = -x^2 + 2x - 2$

4. $y = x^2 + 2x - 3$

Graph the function. State the domain and range.

5. $y = 2x^2 + 8x + 10$

6. $y = -x^2 + 2x + 1$

7. $y = -4x^2 + 32x - 62$

8. $y = 2x^2 + 12x + 19$

Special Cases (Week 3 Assignment #3)**Algebra 1**

Factor.

1. $x^2 + 24x + 144$

2. $y^2 - 14y - 49$

3. $6h^2 + 12h + 6$

4. $x^2 - 121$

5. $10n^2 - 10$

6. $16j^2 - 25k^2$

Solve the equation by factoring.

7. $y^2 - 10y = -25$

8. $49k^2 - 14k + 7 = 6$

9. $x^2 + 4 = 20$

10. $k^2 + 7k + 3 = 2k - 3$

11. $3y^2 = 300$

12. $16y^2 - 36 = 0$

13. $121x^2 - 16 = 0$

14. $5x^2 + 92x + 300 = 12x - 20$

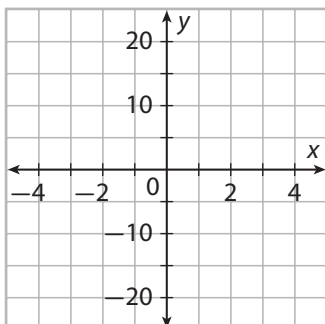
Evaluate: Homework and Practice



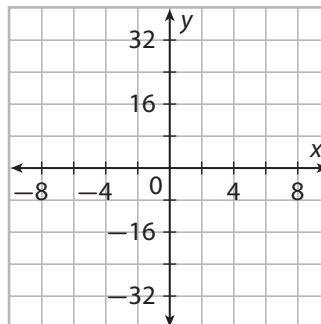
- Online Homework
- Hints and Help
- Extra Practice

Solve each equation by graphing the related function and finding its zeros.

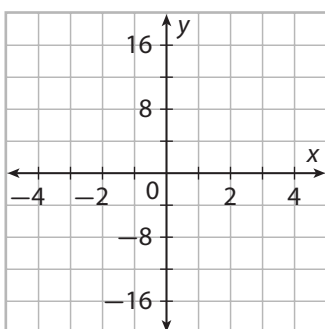
1. $3x^2 - 9 = -6$



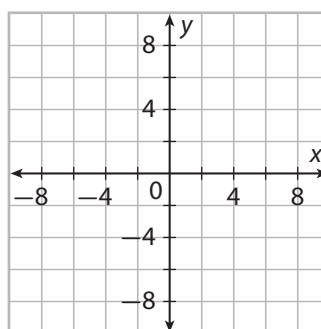
2. $2x^2 - 9 = -1$



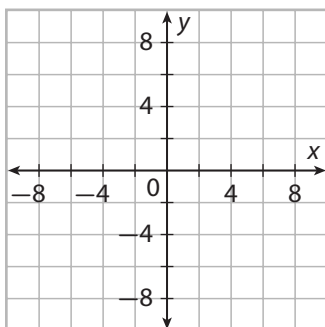
3. $4x^2 - 7 = -3$



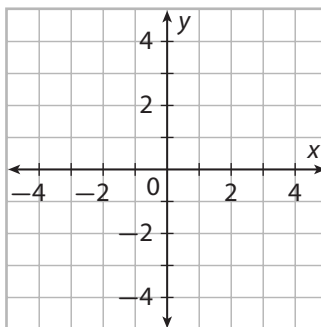
4. $7x + 10 = -x^2$



5. $2x - 3 = -x^2$

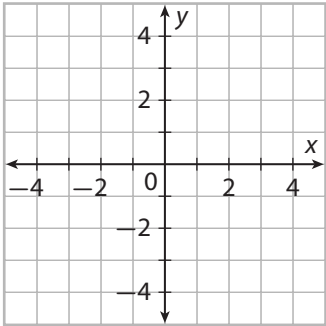


6. $-1 = -x^2$

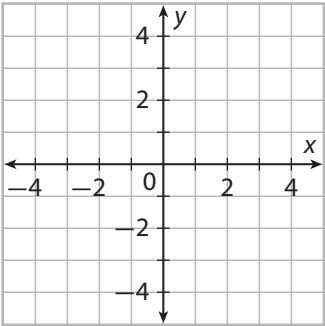


Solve each equation by finding points of intersection of two functions.

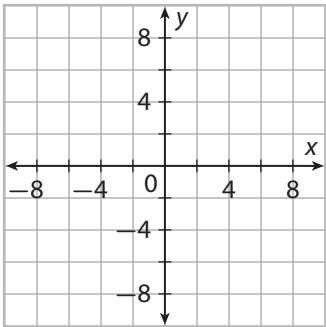
7. $2(x - 3)^2 - 4 = 0$



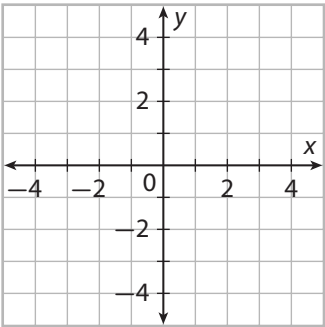
8. $(x + 2)^2 - 4 = 0$



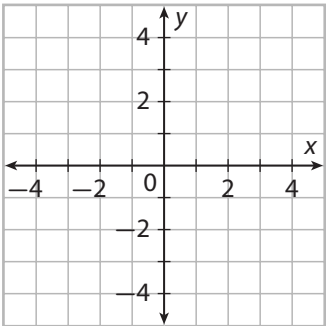
9. $-(x - 3)^2 + 4 = 0$



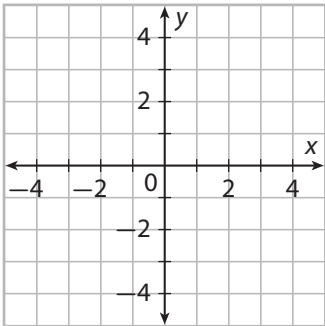
10. $-(x + 2)^2 - 2 = 0$



11. $(x + 1)^2 - 1 = 0$



12. $(x + 2)^2 - 2 = 0$



Write each function in standard form.

5. $y = 5(x - 2)(x + 1)$

6. $y = 2(x + 6)(x + 3)$

7. $y = -2(x + 4)(x - 5)$

8. $y = -4(x + 2)(x + 3)$

9. Which of the following is the correct standard form of $y = 3(x - 8)(x - 5)$?

a. $y = 3x^2 + 39x - 120$

b. $y = x^2 - 13x + 40$

c. $y = 3x^2 - 39x + 120$

d. $y = x^2 - 39x + 40$

e. $y = 3x^2 + 13x + 120$

10. The area of a Japanese rock garden is $y = 7(x - 3)(x + 1)$. Write $y = 7(x - 3)(x + 1)$ in standard form.



Write each function in standard form. Determine x -intercepts and zeros of each function.

11. $y = -(2x - 4)(x - 2)$

12. $y = 2(x + 4)(x - 2)$

13. $y = -3(x + 1)(x - 3)$

14. $y = 2(x + 2)(x - 1)$



Evaluate: Homework and Practice

Find the solutions of each equation.

1. $(x - 15)(x - 22) = 0$

2. $(x + 2)(x - 18) = 0$



- Online Homework
- Hints and Help
- Extra Practice

Find the zeros of each function.

3. $f(x) = (x + 15)(x + 17)$

4. $f(x) = \left(x - \frac{2}{9}\right)\left(x + \frac{1}{2}\right)$

5. $f(x) = -0.2(x - 1.9)(x - 3.5)$

6. $f(x) = x(x + 20)$

7. $f(x) = \frac{3}{4}\left(x - \frac{3}{4}\right)$

8. $f(x) = (x + 24)(x + 24)$



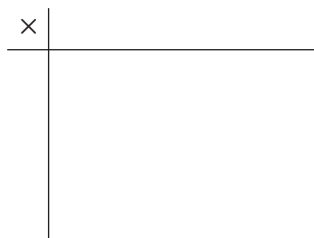
Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

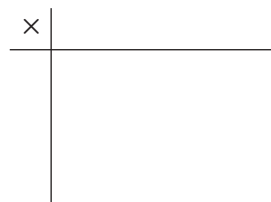
Use algebra tiles to model the factors of each expression.

1. $x^2 + 6x + 8$



$$x^2 + 6x + 8 = (x \boxed{})(x \boxed{})$$

2. $x^2 + 2x - 3$



$$x^2 + 2x - 3 = (x \boxed{})(x \boxed{})$$

Factor the expressions.

3. $x^2 - 15x + 44$

4. $x^2 + 22x + 120$

5. $x^2 + 14x - 32$

6. $x^2 - 12x - 45$

7. $x^2 + 10x + 24$

8. $x^2 + 7x - 8$

Solve each equation.

9. $x^2 + 19x = -84$

10. $x^2 - 18x = -56$

11. $x^2 - 12x + 27 = 0$

12. $x^2 - 9x - 10 = 0$

13. $x^2 + 6x = 135$

14. $x^2 + 13x = -40$

Elaborate

- 9. Discussion** What happens if you do not remove the common factor from the coefficients before trying to factor the quadratic equation?

- 10.** Explain how you can know there are never more than two solutions to a quadratic equation, based on what you know about the graph of a quadratic function.

- 11. Essential Question Check-In** Describe the steps it takes to solve a quadratic equation by factoring.



Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

Factor the following quadratic expressions.

1. $6x^2 + 5x + 1$

2. $9x^2 + 33x + 30$

3. $4x^2 - 8x + 3$

4. $24x^2 - 44x + 12$

5. $3x^2 - 2x - 5$

6. $-10x^2 + 3x + 4$

7. $12x^2 + 22x - 14$

8. $-15x^2 + 21x + 18$

Solve the following quadratic equations.

9. $5x^2 + 18x + 9 = 0$

10. $12x^2 - 36x + 15 = 0$

11. $6x^2 + 28x - 2 = 2x - 10$

12. $-100x^2 + 55x + 3 = 50x^2 - 55x + 23$

13. $8x^2 - 10x - 3 = 0$

14. $-12x^2 = 34x - 28$

15. $(8x + 7)(x + 1) = 9$

16. $3(4x - 1)(4x + 3) = 48x$

Graphing Quadratic Functions in Standard Form

Week 2:
Day 1

Standard form of a Quadratic Equation

$y = ax^2 + bx + c$, where a, b , and c are real numbers and $a \neq 0$.

* When graphing given standard form, will need vertex first like when given vertex form (last week)

⇒ Start with the axis of symmetry (the x-coordinate of the vertex!)

The axis of symmetry for a quadratic equation in standard form is given by the equation $x = \frac{-b}{2a}$

The vertex of a quadratic equation in standard form is

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$$

↑
axis of
symmetry
formula

↑ after finding the x-coordinate, plug that value into the equation to find the y-value (like we do with any other point)

Ex give the axis of symmetry and the coordinates of the vertex of the quadratic equation

a $y = 2x^2 + 8x + 12$

Axis of Symmetry $\Rightarrow x = \frac{-b}{2a}$

$a = 2$ $b = 8$

$x = \frac{-8}{2(2)} = \frac{-8}{4}$

$x = -2$ Axis of Symmetry

b $y = x^2 - 12x - 2$

$a = 1$ $b = -12$

Axis of Symmetry $x = \frac{-(-12)}{2(1)} = \frac{12}{2}$
 $x = 6$

c $y = -4x^2 + 12$

$a = -4$ $b = 0$

Axis of Symmetry

$x = \frac{-0}{2(-4)} = \frac{0}{-8}$

$x = 0$

from axis of symmetry!
Vertex $(-2, ?)$ to find y, plug x (-2) into equation!

$y = 2(-2)^2 + 8(-2) + 12$

$y = 2(4) - 16 + 12$

$= 8 - 16 + 12$

$= -8 + 12$

$y = 4$

★ Vertex is $(-2, 4)$

Vertex is $(6, ?)$

$y = (6)^2 - 12(6) - 2$

$= 36 - 72 - 2$

$= -36 - 2$

$y = -38$

★ Vertex is $(6, -38)$

Vertex is $(0, ?)$

$y = -4(0)^2 + 12$

$= -4(0) + 12$

$= 0 + 12$

$y = 12$

★ Vertex is $(0, 12)$

How to Graph a Quadratic function in Standard form

- 1) Find the vertex (x-coordinate is $x = \frac{-b}{2a}$, then plug into equation to find y-coordinate)
- 2) Make a table of points with 2 points on either side of the vertex
- 3) Plot 5 points from table and draw parabola

Ex Graph the function

a $y = 2x^2 - 16x + 30$

Vertex: $a=2$ $b=-16$

$x = \frac{-(-16)}{2(2)} = \frac{16}{4} = 4$

$y = 2(4)^2 - 16(4) + 30$
 $= 2(16) - 64 + 30$
 $= 32 - 64 + 30$
 $= -32 + 30$
 $y = -2$

Vertex: $(4, -2)$

x	y
2	6
3	0
4	-2
5	0
6	6

Vertex \rightarrow 4

} found by symmetry (less to plug in!)

$x=2$

$y = 2(2)^2 - 16(2) + 30$

$2(4) - 32 + 30$

$8 - 32 + 30$

$-24 + 30$

$y = 6$

$x=3$

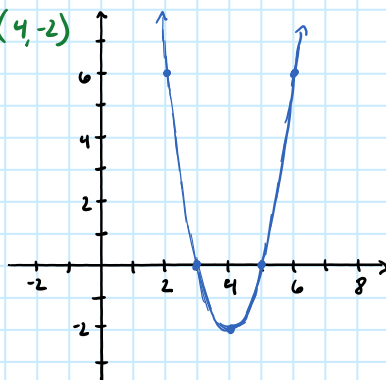
$y = 2(3)^2 - 16(3) + 30$

$= 2(9) - 48 + 30$

$= 18 - 48 + 30$

$= -30 + 30$

$y = 0$



b $y = -x^2 - 4x$

Vertex: $a=-1$ $b=-4$

$x = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2$

$y = -(-2)^2 - 4(-2)$
 $= -(4) + 8$
 $= -4 + 8$
 $= 4$

Vertex: $(-2, 4)$

x	y
-4	0
-3	3
-2	4
-1	3
0	0

Vertex \rightarrow -2

← using symmetry again!

$x=-1$

$y = -(-1)^2 - 4(-1)$

$= -(1) + 4$

$= -1 + 4$

$= 3$

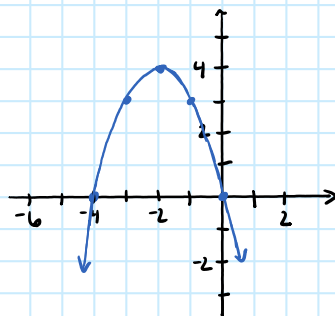
$x=0$

$y = -(0)^2 - 4(0)$

$= -(0) - 0$

$= 0 - 0$

$= 0$



Solve Quadratic Equations Graphically

Week 2:
Day 2-3

zero of a function - an x -value that makes the value of a function 0.

★ zeros of a function are the x -intercepts of the function's graph

A quadratic function can have 0, 1, or 2 zeros



Solving Quadratic Equations by Graphing

- rewrite equation so it is $=0$ (if necessary)
 - replace 0 with y and graph the equation (look at previous notes, depending on form in!)
 - Find the x -intercepts, which are the zeros of the function
- ★ since looking for when equation $=0$, these x -intercepts (zeros) are the solutions of the original equation

Ex Solve by graphing the related functions

$$\begin{aligned} & \underline{a} \quad 3x^2 + 2 = 5 \\ & \quad \quad \quad -5 \quad -5 \quad \star \text{ make } = 0 \\ & 3x^2 - 3 = 0 \end{aligned}$$

$$y = 3x^2 - 3 \quad \star \text{ make 0 into } y \text{ \& graph function}$$

↑ standard form
so find vertex using $x = \frac{-b}{2a}$

$$\begin{aligned} a &= 3 \quad b = 0 \\ x &= \frac{-0}{2(3)} = \frac{0}{6} = 0 \end{aligned}$$

$$\begin{aligned} y &= 3(0)^2 - 3 \\ &= 3(0) - 3 \\ &= 0 - 3 \\ y &= -3 \end{aligned}$$

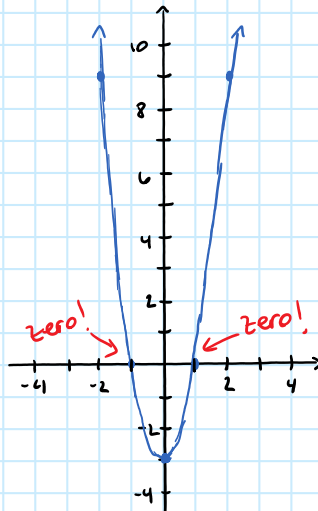
Vertex is $(0, -3)$

x	y
-2	9
-1	0
0	-3
1	0
2	9

$$\begin{aligned} x &= -2 \\ y &= 3(-2)^2 - 3 \\ &= 3(4) - 3 \\ &= 12 - 3 \\ &= 9 \end{aligned}$$

$x=2$ is the same by symmetry

$$\begin{aligned} x &= 1 \\ y &= 3(1)^2 - 3 \\ &= 3(1) - 3 \\ &= 3 - 3 \\ y &= 0 \\ x &= -1 \text{ is the same by symmetry} \end{aligned}$$



★ Since $x = -1$ and $x = 1$ are the zeros (x -int) & the equation $= 0$ they are the solutions!

$$\underline{x = -1, 1}$$

b $6x+8 = -x^2$

$+x^2 \quad +x^2$
 $x^2+6x+8=0$

★ make = 0
 and replace 0 with y

$y = x^2+6x+8$ ★ in standard form so use $x = \frac{-b}{2a}$ to find vertex

Vertex: $x = \frac{-6}{2(1)} = -\frac{6}{2} = -3$

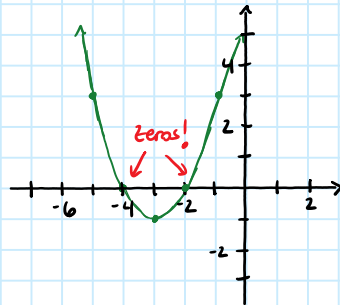
$y = (-3)^2 + 6(-3) + 8$
 $= 9 - 18 + 8$
 $= -9 + 8$
 $y = -1$

Vertex: $(-3, -1)$

x	y
-5	3
-4	0
-3	-1
-2	0
-1	3

$x=5$
 $y = (-5)^2 + 6(-5) + 8$
 $= 25 - 30 + 8$
 $= -5 + 8$
 $= 3$
 ★ also -1 by symmetry

$x=-4$
 $y = (-4)^2 + 6(-4) + 8$
 $= 16 - 24 + 8$
 $= -8 + 8$
 $= 0$
 ★ also $x=-2$ by symmetry



★ zeros (x-int) are at -2 and -4
 and since looking for when the equation is = 0, they are the solutions!
 $x = -2$ and $x = -4$

c $2(x-3)^2 - 2 = 0$

★ already = 0 so just replace 0 with y

$y = 2(x-3)^2 - 2$

★ In Vertex form $y = a(x-h)^2 + k$, So Vertex is (h, k) , no work to find it!!

Vertex: $(3, -2)$

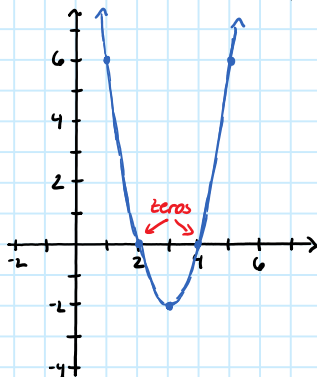
x	y
1	6
2	0
3	-2
4	0
5	6

$x=1$
 $y = 2(1-3)^2 - 2$
 $= 2(2)^2 - 2$
 $= 2(4) - 2$
 $= 8 - 2$
 $y = 6$

★ also have $x=5$
 by symmetry!

$x=4$
 $y = 2(4-3)^2 - 2$
 $= 2(1)^2 - 2$
 $= 2(1) - 2$
 $= 2 - 2$
 $y = 0$

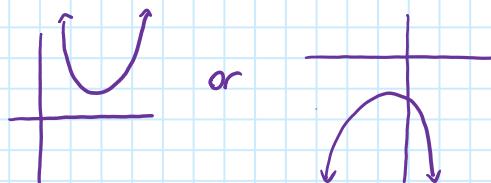
★ also have $x=2$
 by symmetry!



★ zeros are at 2 and 4 Since looking for when equation is equal to 0 they are the solutions!!
 $x = 2$ and $x = 4$

★ What happens if the graph looks like where there are no zeros?

⇒ Then there is No Solution!



★ Remember Quadratic equations can have 2, 1, or No Solutions!

Factored Form of a Quadratic Equation

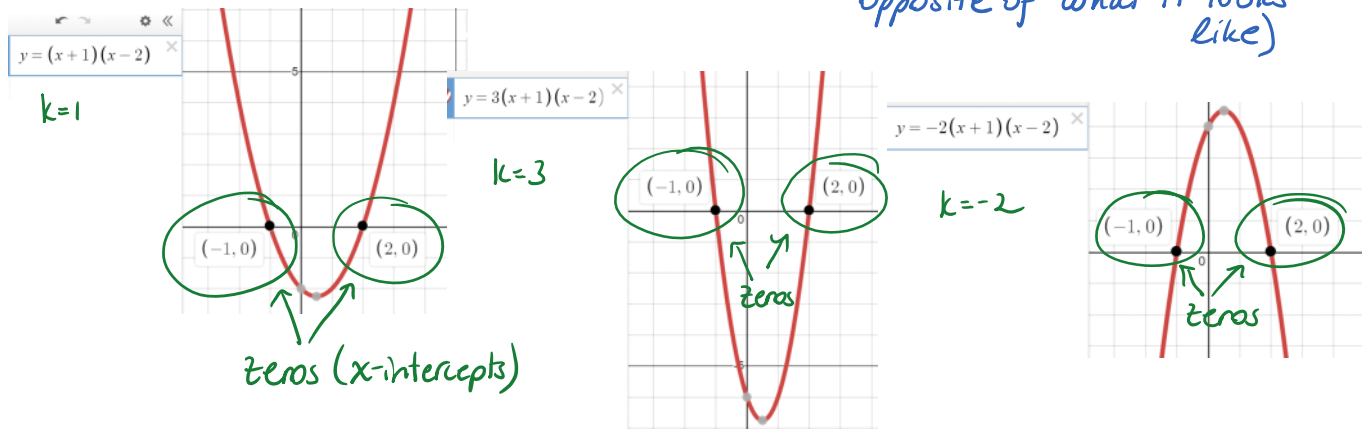
Week 2:
Day 4

Factored form of a quadratic equation

$$y = k(x-a)(x-b) \quad \text{where } k \neq 0$$

★ Let's look at the graphs of $y = k(x+1)(x-2)$ for $k=1, 3$, and -2

⇒ so in this case $a = -1$ and $b = 2$ (they are like h in Vertex form opposite of what it looks like)



★ Notice that the x-intercepts (zeros) are -1 and 2

(the a & b values, what are be subtracted from x in each $()$!)

k does not change the zeros, just stretches the graph or flips it over

So, the factors (the parentheses) in factored form give us the x-intercepts (zeros)

★ in $y = k(x-a)(x-b)$ a & b are the x-intercepts (zeros) of the function!

Ex Write each function in standard form. Determine the x-intercepts & zeros of each function

a $y = 4(x-5)(x-1)$

★ Let's find x-intercepts (zeros) first since easy in current form (factored form)

Since $a=5$ and $b=1$

The x-intercepts & zeros are $x=5$ & $x=1$

★ To write in standard form, multiply 2 items then that answer with the remaining item

$$y = 4(x-5)(x-1)$$

	x	-5	
x	x^2	$-5x$	
-1	$-x$	5	

$$x^2 - 6x + 5$$

$$y = 4(x^2 - 6x + 5)$$

$$y = 4x^2 - 24x + 20$$

b $y = -3(x+4)(x+3)$

x-intercepts & zeros are

$$x = -4 \text{ \& } x = -3$$

★ Since

$$y = -3(x+4)(x+3)$$

\downarrow \downarrow
 $x - (-4)$ $x - (-3)$
 \uparrow
 x-intercepts!

★ Standard form

$$y = -3(x+4)(x+3)$$

$$(x+4)(x+3)$$

$$x^2 + 3x + 4x + 12$$

$$= x^2 + 7x + 12$$

$$y = -3(x^2 + 7x + 12)$$

$$y = -3x^2 - 21x - 36$$

Using Zero Product Property to Solve Equations

Week 2:
Day 5

Zero Product Property

For all real numbers a and b , if the product of the two quantities equals zero, then at least one of the quantities equals zero

If $ab=0$, then $a=0$ or $b=0$

★ The only way to get zero when multiplying is for 1 of the items being multiplied to be 0!

Ex Find the zeros of each function ★ meaning find x values that make the function $=0$!

a $f(x) = (x-16)(x+21)$

★ replace $f(x)$ or y with 0 since looking for zeros!

$$0 = (x-16)(x+21)$$

★ Apply Zero Product Property (since 2 things multiplied to get 0!)
meaning either $1^{st}()=0$ or $2^{nd}()=0$

$$\begin{array}{cc} x-16=0 & \text{or} & x+21=0 \\ +16 & +16 & -21 & -21 \end{array}$$

$$x=16 \text{ or } x=-21$$

★ now solve new equations!

b $g(x) = -4(x-8)$ ★ replace $g(x)$ with 0

$$0 = -4(x-8)$$

★ apply Zero Product Property

$$\begin{array}{cc} -4=0 & x-8=0 \\ \uparrow & +8 \quad +8 \\ \text{not relevant} & \\ \text{since no} & \\ \text{variable!} & \end{array}$$

$$x=8$$

c $h(x) = 3x(x-12)$

$$0 = 3x(x-12)$$

$$\begin{array}{cc} 3x=0 & x-12=0 \\ \div 3 & +12 \quad +12 \end{array}$$

$$x=0 \text{ or } x=12$$

d $k(x) = \frac{1}{2}(x-2)(x+3)$

$$0 = \frac{1}{2}(x-2)(x+3)$$

★ only care about factors that have variable!
(so ignore $\frac{1}{2}$)

$$\begin{array}{cc} x-2=0 & x+3=0 \\ +2 \quad +2 & -3 \quad -3 \end{array}$$

$$x=2 \text{ or } x=-3$$

Factoring Trinomials

Week 3:

Day 1-2

factoring - the process of writing a polynomial as a product

⇒ like Jeopardy style questions of multiplying

What was multiplied to get $3y+6$ or $x^2+5x-14$?

$$\begin{array}{c} 3y+6 \\ \downarrow \\ 3(y+2) \end{array}$$

$$\begin{array}{c} x^2+5x-14 \\ \downarrow \\ (x+7)(x-2) \end{array}$$

Factor out a common factor

Ex $4x^2+26x+42$

★ look for the greatest common factor (GCF) of the three terms ⇒ meaning find the largest number that goes into the coefficients 4, 26, 42

★ one way is to list all numbers (factors) of the #'s and identify the largest

GCF of 4, 26, and 42 is 2.

So can rewrite

$$2(\text{?} + \text{?} + \text{?})$$

★ to find what goes in (?) ask what multiplied by 2 (the GCF) gets each term

$$\begin{array}{c} 2(\text{?})=4x^2 \\ \downarrow \\ 2x^2 \end{array}$$

$$\begin{array}{c} 2(\text{?})=26x \\ \downarrow \\ 13x \end{array}$$

$$\begin{array}{c} 2(\text{?})=42 \\ \downarrow \\ 21 \end{array}$$

$$\text{Or } \frac{4x^2}{2} + \frac{26x}{2} + \frac{42}{2}$$

$$2(2x^2+13x+21)$$

★ GCF of a polynomial can contain variables, but in order for that to happen all the terms must have a variable

Ex $6x^3-21x^2-45x$

★ All 3 terms have at least $1x$ so GCF has an x , Next find GCF of 6, -21, -45

GCF of 6, -21, -45 is 3 so can "pull out" $3x$

$$3x(2x^2-7x-15)$$

$$3x(\text{?})=6x^3 \text{ or } \frac{6x^3}{3x}$$

$$3x(\text{?})=-21x^2 \text{ or } \frac{-21x^2}{3x}$$

$$3x(\text{?})=-45x \text{ or } \frac{-45x}{3x}$$

★ You can easily check your answer by multiplying out your answer to see if you get the question!!

★★★ Always pull out any common factors before factoring polynomials!!!

Factor ax^2+bx+c → Use box & X method

→ Before we factor let's remember multiplying with the box (area model)

$$(x+7)(x-2)$$

	x	7
x	x^2	$7x$
-2	$-2x$	-14

$x^2+5x-14$
 ↑ top left square
 ↑ bottom right square
 combinations of other 2 boxes

$$(5x-2)(2x+3)$$

	$5x$	-2
$2x$	$10x^2$	$-4x$
3	$15x$	-6

$10x^2+11x-6$
 ↑ top left box
 ↑ bottom right square
 combinations of other 2 boxes

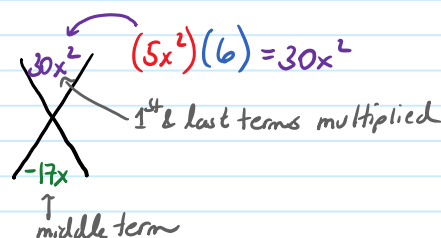
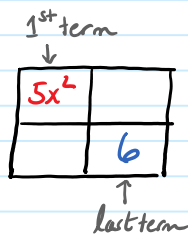
★ 1st term in answer comes from multiplying 1st terms of binomials

★ last term in answer comes from multiplying 2nd terms of binomials

★ Middle term is a combinations of the terms in the binomials (this is where we need to find the right combination!)

Ex Factor $5x^2$ - $17x$ + 6

- 1) Fill in 4 spots of the box and X



- 2) Looking for 2 items that multiply to make the top of the X (in our example $30x^2$) and add up to make the bottom of the X (in our example $-17x$)

→ since multiply to be x^2 and add up to x , both have an x !
Now just to find 2 #'s that multiply to make $+30$ and add up to -17

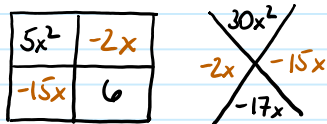
1 way is to list all pairs that multiply 30

$-1 -30$
 $-2 -15$
 $-3 -10$
 $-5 -6$

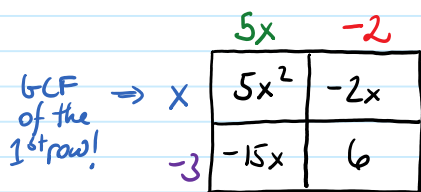
★ since add up a negative & multiply to be a positive both are -

⇒ Only pair that adds up to -17 is -2 & -15

So $-2x$ & $-15x$ goes in the 4 remaining empty spots



- 3) Now need to find the binomials that made this box!



★ After finding that 1st item use it to find the remaining 3 items

$x(?) = 5x^2$ $x(?) = -2x$ $5x(?) = -15x$

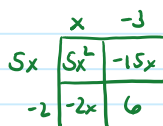
\downarrow \downarrow \downarrow

$5x$ -2 -3

★ Nice check built in, can see that $-2(-3) = 6$ so we did something right!

- 4) Write out binomials from box

$(5x-2)(x-3)$



A few notes...

→ $-2x$ & $-15x$ could have been switched in the box (still gets same binomials!)

→ It doesn't matter the order of the $()$ in the answer only what is inside the $()$ matches
So could write answer as $(x-3)(5x-2)$

Ex Factor

a $x^2 + 7x + 6$

★ Can we pull out something?

→ No since no variable in last term and 1st term's coefficient is 1 (GCF = 1 so nothing to pull out)

★ Now factor trinomial

	x	6
x	x^2	$6x$
1	$1x$	6

	$6x^2$
$1x$	$6x$
	$7x$

$$\begin{array}{r} 6x^2 \\ 1x \quad 6x \Rightarrow 7x \\ 2x \quad 3x \Rightarrow 5x \end{array}$$

$(x+1)(x+6)$

★ x^2 & $6x$ (top row) GCF is x

$-x(?) = x^2$ need x } top binomial

$-x(?) = 6x$ need 6 }

$-x(?) = x$ need 1 ← side/bottom

$1(6) = 6$ so fits box

b $8x^2 - 18x - 5$

★ Can we pull something out?

→ No since no variable in last term and GCF of 8, 18, and 5 is 1

★ Now factor trinomial

	$4x$	1
$2x$	$8x^2$	$2x$
-5	$-20x$	-5

	$-40x^2$
$2x$	$-20x$
	$-18x$

$$8x^2(-5) = -40x^2$$

sum is

$$-4x \quad 10x \Rightarrow 6x$$

$$-2x \quad 20x \Rightarrow 18x$$

$$2x \quad -20x \Rightarrow -18x$$

★ Right # wrong sign!

so just switch signs of numbers!

$(4x+1)(2x-5)$

★ $8x^2$ & $2x$ (top row) GCF is $2x$

$2x(?) = 8x^2$ need $4x$ } top binomial

$2x(?) = 2x$ need 1 }

$4x(?) = -20x$ need -5

$1(-5) = -5$ so fits box

c $x^2 - 11x + 30$

★ Can we pull something out? No

★ Now factor trinomial

	x	-5
x	x^2	$-5x$
-6	$-6x$	30

	$30x^2$
$-5x$	$-6x$
	$-11x$

$$\begin{array}{r} 30x^2 \\ 3x \quad 10x \Rightarrow 13x \\ -5x \quad -6x \Rightarrow -11x \end{array}$$

$(x-5)(x-6)$

d $2x^2 + 7x - 9$

★ Can we pull something out? No. Now factor trinomial

	x	-1
$2x$	$2x^2$	$-2x$
9	$9x$	-9

	$-18x^2$
$-2x$	$9x$
	$7x$

$$2x^2(-9) = -18x^2$$

sum is

$$2x \quad -9x \Rightarrow -7x$$

$$-2x \quad 9x \Rightarrow 7x$$

$(2x+9)(x-1)$

e $15x^2 - 42x - 9$

★ Can we pull something out? GCF of 15, 42, and 9 is 3 so Yes!

$3(5x^2 - 14x - 3)$ $\frac{15x^2}{3} = 5x^2$ $\frac{-42x}{3} = -14x$ $\frac{-9}{3} = -3$

★ Now factor trinomial in ()

	x	-3
$5x$	$5x^2$	$-15x$
1	$1x$	-3

$\begin{array}{r} -15x^2 \\ 1x \end{array} \begin{array}{r} -15x \\ -14x \end{array}$

$5x^2(-3) = -15x^2$
 $3x - 5x \Rightarrow -2x$
 $1x - 15x \Rightarrow -14x$

$3(5x+1)(x-3)$

★ Remember to include the GCF, that we pulled out, in front of the binomials

f $36x^2 + 72x + 20$

★ GCF is 4

$4(9x^2 + 18x + 5)$

★ Now trinomial

	$3x$	5
$3x$	$9x^2$	$15x$
1	$3x$	5

$\begin{array}{r} 45x^2 \\ 15x \end{array} \begin{array}{r} 15x \\ 3x \end{array}$

$9x^2(5) = 45x^2$
 $9x - 5x \Rightarrow 4x$
 $15x - 3x \Rightarrow 12x$

$4(3x+5)(3x+1)$

g $2x^2 + 26x - 60$

★ GCF is 2

$2(x^2 + 13x - 30)$

	x	-2
x	x^2	$-2x$
15	$15x$	-30

$\begin{array}{r} -30x^2 \\ -2x \end{array} \begin{array}{r} 15x \\ 13x \end{array}$

$-30x^2$
 $-3x - 10x \Rightarrow -13x$
 $-5x - 6x \Rightarrow -11x$
 $-2x - 15x \Rightarrow -17x$

$2(x-2)(x+15)$

Solving Quadratic Equations by Factoring

Week 3:
Day 3-4

By factoring we can write a quadratic equation in factored form, then use the zero product property to solve the equation.

★ Make the equation $= 0$ BEFORE factoring (so can apply zero product property after factoring)

⇒ Keep/make x^2 term positive (if x^2 term is negative, factor out a -1 to make the x^2 term positive)

Ex Solve each equation by factoring

a $x^2 - 7x = -10$

$x^2 - 7x + 10 = 0$ ★ Make $= 0$

★ factor nonzero side (No GCF, so onto box and X)

	x	-2
x	x^2	$-2x$
-5	$-5x$	10

$10x^2$	
$-2x$	$-5x$
	$-7x$

$\frac{10x^2}{-2x - 5x} \Rightarrow -7x$

$(x-2)(x-5) = 0$

★ Apply Zero Product Property (set each factor, $()$, $= 0$)

$x-2=0$ $x-5=0$

$+2$ $+2$ $+5$ $+5$ ★ Solve the equations

$x=2$ or $x=5$

b $4x^2 - 4x - 3 = 0$

★ Already $= 0$, so factor nonzero side

No GCF > 1 so on to box and X

	$2x$	-3
$2x$	$4x^2$	$-6x$
1	$2x$	-3

$-12x^2$	
$-6x$	$2x$
	$-4x$

$\frac{-12x^2}{-4x \quad 3x \Rightarrow -1x}$
 $\frac{-6x \quad 2x \Rightarrow -4x}$

$(2x+1)(2x-3) = 0$

★ Apply Zero Product Property

$2x+1=0$ $2x-3=0$

$\frac{2x}{2} = \frac{-1}{2}$

$\frac{2x}{2} = \frac{3}{2}$

★ Solve the equations

$x = -\frac{1}{2}$ or $x = \frac{3}{2}$

$$c) \quad \begin{array}{cc} x^2 - 14 = -5x \\ +5x \quad +5x \end{array}$$

★ Make = 0, make sure trinomial is in standard form

$$x^2 + 5x - 14 = 0$$

★ Factor nonzero side, No GCF > 1 so use \boxtimes

	x	7
x	x^2	$7x$
-2	$-2x$	-14

	$-14x^2$
$7x$	$-2x$
$5x$	

$$\begin{array}{l} \frac{-14x^2}{-7x \quad 2x} \Rightarrow -5x \\ \frac{-14x^2}{7x \quad -2x} \Rightarrow 5x \end{array}$$

$$(x+7)(x-2) = 0$$

$$\begin{array}{cc} x+7=0 & x-2=0 \\ -7 & +2 \end{array}$$

$$x = -7 \text{ or } x = 2$$

★ Apply Zero Product Property and solve resulting equations

$$d) \quad \begin{array}{cc} 12x^2 + 48x = -45 \\ +45 \quad +45 \end{array}$$

★ Make = 0 and factor nonzero side

$$\frac{12x^2}{3} + \frac{48x}{3} + \frac{45}{3} = 0$$

12, 48, 45 has GCF of 3, so pull out GCF then use \boxtimes

$$3(4x^2 + 16x + 15) = 0$$

	$2x$	3
$2x$	$4x^2$	$6x$
5	$10x$	15

	$60x^2$
$6x$	$10x$
	$16x$

$$\begin{array}{l} \frac{60x^2}{4x \quad 15x} \Rightarrow 15x \\ \frac{60x^2}{3x \quad 20x} \Rightarrow 20x \\ \frac{60x^2}{6x \quad 10x} \Rightarrow 10x \end{array}$$

$$3(2x+5)(2x+3) = 0$$

$$\begin{array}{cc} 2x+5=0 & 2x+3=0 \\ -5 & -3 \end{array}$$

$$\frac{2x}{2} = \frac{-5}{2} \quad \frac{2x}{2} = \frac{-3}{2}$$

$$x = -\frac{5}{2} \text{ or } x = -\frac{3}{2}$$

★ Apply Zero Product property and solve resulting equations

⇒ Remember the factors without a variable (the GCF) is not relevant when solving the equation

$$e) \quad \begin{array}{cc} x^2 - x = 56 \\ -56 \quad -56 \end{array}$$

★ make = 0 and factor nonzero side

$$x^2 - x - 56 = 0$$

No GCF so \boxtimes

	x	-8
x	x^2	$-8x$
7	$7x$	-56

	$-56x^2$
$-8x$	$7x$
$-x$	

$$\frac{-56x^2}{7x \quad -8x} \Rightarrow -x \checkmark$$

$$(x+7)(x-8) = 0$$

★ Apply Zero Product Property

$$\begin{array}{cc} x+7=0 & x-8=0 \\ -7 & +8 \end{array}$$

$$x = -7 \text{ or } x = 8$$

f $2x^2 + 7x - 2 = 4x^2 + 4$ \star make $= 0$ (keep x^2 positive, so move every thing to the right side)

$0 = 2x^2 - 7x + 6$ \star No GCF, so use $\boxtimes X$

	$2x$	-3
x	$2x^2$	$-3x$
-2	$-4x$	6

	$12x^2$
$-3x$	$-4x$
$-7x$	

$12x^2$	
$-2x$	$-6x \Rightarrow -8x$
$-3x$	$-4x \Rightarrow -7x$

$(x-2)(2x-3)=0$

$x-2=0$ $2x-3=0$
 $+2 +2$ $+3 +3$
 $\frac{2x}{2} = \frac{3}{2}$
 $x=2$ or $x=\frac{3}{2}$

\star Apply Zero Product Property & solve equations

g $3(x^2-1) = -3x^2 + 2x + 5$
 $3x^2 - 3 = -3x^2 + 2x + 5$
 $+3x^2 - 2x - 5 +3x^2 - 2x - 5$

\star Make $= 0$, get rid of $()$ then move terms
 \Rightarrow keeping x^2 term positive!

$6x^2 - 2x - 8 = 0$
 $2(3x^2 - x - 4) = 0$

\star GCF of 6, 2, 8 is 2 so factor out then use $\boxtimes X$

	x	1
$3x$	$3x^2$	$3x$
-4	$-4x$	-4

	$12x^2$
$3x$	$-4x$
$-x$	

$-12x^2$	
$3x$	$-4x \Rightarrow -x$

$(3x-4)(x+1)=0$

\star Apply Zero Product Property

$3x-4=0$ $x+1=0$
 $+4 +4$ $-1 -1$

$\frac{3x}{3} = \frac{4}{3}$

$x = \frac{4}{3}$ or $x = -1$

Using Special Factors to Solve Equations

Week 3:
Day 5

Perfect Square Trinomials

⇒ Trinomials when factored result in the binomials being the same
factor them like any other trinomials!

Ex $x^2 + 10x + 25$

	x	5
x	x^2	$5x$
5	$5x$	25

$\begin{array}{r} 25x^2 \\ 5x \quad 5x \\ \hline 10x \end{array}$

$\frac{25x^2}{5x} = 5x \Rightarrow 10x$

$= (x+5)(x+5)$
or $= (x+5)^2$

★ Remember pull out common factor first if possible (can be a number and variable)

Ex Factor

a $3x^3 - 24x^2 + 48x$

all terms have at least 1 x & GCF of 3, 24, 48 is 3
so GCF = 3x

$3x(x^2 - 8x + 16)$

$\frac{3x^3}{3x} = x^2$

$\frac{-24x^2}{3x} = -8x$

$\frac{48x}{3x} = 16x$

	x	-4
x	x^2	$-4x$
-4	$-4x$	16

$\begin{array}{r} 16x^2 \\ -4x \quad -4x \\ \hline -8x \end{array}$

$\frac{16x^2}{-4x} = -4x$
 $\frac{-24x^2}{-4x} = -8x$
 $\frac{48x}{-4x} = -12$

$3x(x-4)^2$

OK to write as
 $3x(x-4)(x-4)$

★ Remember to include the GCF that was pulled out.

b $100y^2 - 20y + 1$

★ No GCF > 1, so on to \boxtimes

	$10y$	-1
$10y$	$100y^2$	$-10y$
-1	$-10y$	1

$\begin{array}{r} 100y^2 \\ -10y \quad -10y \\ \hline -20y \end{array}$

$\frac{100y^2}{-10y} = -10y$
 $\frac{-20y}{-10y} = 2$

$(10y-1)^2$

★ OK if write $(10y-1)(10y-1)$

Difference of Two Squares

$$a^2 - b^2 = (a-b)(a+b)$$

★ if see a binomial, then possibly a difference of two squares

⇒ is it subtraction?

⇒ are both terms perfect squares?

Ex Factor

c $x^2 - 64$

★ 2 terms & subtraction

$(x)^2 - (8)^2$ → both terms can be written as squares so difference of 2 squares!

$$(x+8)(x-8)$$

d $25r^2 - 49p^2$

★ No GCF

★ 2 terms & subtraction, is it a difference of two squares?

$$(5r)^2 - (7p)^2$$

Yes! so $(a+b)(a-b)$
 $a=5r$ $b=7p$

$$(5r+7p)(5r-7p)$$

e $81y^4 - 9y^2$

★ GCF! $9y^2$, so pull out

$$9y^2(9y^2 - 1)$$

$$(3y)^2 - (1)^2$$

★ Factor (), 2 terms & subtracting so are the 2 terms perfect squares?
Yes, so factor!

$$9y^2(3y-1)(3y+1)$$

f $16x^4 - 1$

★ No GCF, 2 terms with subtraction so are both terms perfect squares?

$$(4x^2)^2 - (1)^2$$

$x^4 = x^2 \cdot x^2$ or $(x^2)^2$!

$$(4x^2-1)(4x^2+1)$$

$$(2x)^2 - (1)^2$$

★ Notice the first () has an exponent on x & subtraction so need to check if difference of two squares
It is! so factor it!

$$(2x+1)(2x-1)(4x^2+1)$$

Remember When solving an equation by factoring

★ make = 0 if not already

★ Factor non zero side

★ Use the Zero Product Property to solve equation

Ex Solve by factoring

a $4x^2 + 12x + 9 = 0$ ★ = 0 already, so factor

★ No GCF, has 3 terms so ~~box~~ X

	2x	3
2x	4x ²	6x
3	6x	9

~~$$\begin{array}{cc} 36x^2 & \\ 6x & 6x \\ & 12x \end{array}$$~~

$$\frac{36x^2}{6x} = 6x \Rightarrow 12x$$

$$(2x+3)^2 = 0$$

$$2x+3=0$$

$$\frac{2x}{2} = \frac{-3}{2}$$

$$x = -\frac{3}{2}$$

★ Apply Zero Product Property, since both binomials are the same only need 1 equation

b $25x^2 - 1 = 0$

★ already = 0, no GCF so factor

⇒ 2 terms with subtraction, so difference of two squares?

$$(5x)^2 - (1)^2$$

$$(5x+1)(5x-1) = 0$$

★ Apply Zero Product Property

$$\begin{array}{cc} 5x+1=0 & 5x-1=0 \\ \frac{5x}{5} = \frac{-1}{5} & \frac{5x}{5} = \frac{1}{5} \end{array}$$

$$x = -\frac{1}{5} \text{ or } x = \frac{1}{5}$$

c $8x^4 - 2x^2 = 0$

★ already = 0, has GCF of $2x^2$, so factor out

$$2x^2(4x^2 - 1) = 0$$

$$(2x)^2 - 1^2$$

★ () has 2 terms and subtraction, so difference of two squares?

$$2x^2(2x+1)(2x-1) = 0$$

★ Apply Zero Product Property, since all 3 factors have variables, 3 equations are necessary

$$\frac{2x^2}{2} = \frac{0}{2}$$

$$x^2 = 0$$

$$x = 0$$

$$2x+1=0$$

$$\frac{2x}{2} = \frac{-1}{2}$$

$$x = -\frac{1}{2}$$

$$2x-1=0$$

$$\frac{2x}{2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

★ $x^2 = 0 \Rightarrow$ only $\sqrt{}$ to square and get 0 is 0!

★ Not quadratic (has x^4) so ok that has more than 2 solutions!